9.2

$\boxed{1}$ $\frac{3}{7}+\frac{3}{7}^2+\cdots+\left(\frac{3}{7}\right)^{100}=\frac{3}{7}+\frac{3}{7}\left(\frac{3}{7}\right)+\cdots+\frac{3}{7}\left(\frac{3}{7}\right)^{99}$

$a=\frac{3}{7}$, $x=\frac{3}{7}$, $n-1=99 \Rightarrow n=100$

$$\frac{a(1-x^n)}{1-x}=\frac{\frac{3}{7}\left(1-\left(\frac{3}{7}\right)^{100}\right)}{1-\frac{3}{7}}=\frac{\frac{3}{7}\left(1-\left(\frac{3}{7}\right)^{100}\right)}{\frac{4}{7}}$$

$$=\frac{3}{4}\left(1-\left(\frac{3}{7}\right)^{100}\right)$$

$\boxed{2}$ $e^{1/2}\cdot e^{1/4}\cdot e^{1/8}\cdots=e^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots}$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots=\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{2}\right)^2+\cdots$

$a=\frac{1}{2}$, $x=\frac{1}{2}$

$$=e^{\frac{\frac{1}{2}}{1-\frac{1}{2}}}=e^1=e$$

$\boxed{3}$

Total Spending (in millions) $=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$

$a=1$, $x=\frac{1}{2}$

$$=\frac{1}{1-\frac{1}{2}}=2 \text{ million dollars}$$

<u>10.1</u>

$\boxed{1}$ $f(x) = \arctan x$ $\quad f(0) = 0$

$f'(x) = \dfrac{1}{1+x^2}$ $\quad f'(0) = 1$

$f''(x) = \dfrac{-2x}{1+x^2}$ $\quad f''(0) = 0$

$f'''(x) = \dfrac{2x^2-2}{(1+x^2)^2}$ $\quad f'''(0) = -2$

$$\arctan x \approx x - \frac{2x^3}{3!}$$

$$\arctan(0.25) \approx .25 - \frac{2(.25)^3}{3!} = .24479$$

$\arctan(0.25)$ & the estimate agree to three decimal places.

$\boxed{2}$ $f(x) \approx 4 + 3(x-2) - \dfrac{5(x-2)^2}{2!} + \dfrac{12(x-2)^3}{3!}$

$\boxed{3}$ $f(x) = \sqrt{1-x}$ $\quad f(0) = 1$

$f'(x) = \dfrac{-1}{2\sqrt{1-x}}$ $\quad f'(0) = -\dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1-x)^{3/2}}$ $\quad f''(0) = -\dfrac{1}{4}$

$f'''(x) = \dfrac{-3}{8(1-x)^{5/2}}$ $\quad f'''(0) = -\dfrac{3}{8}$

$$\sqrt{1-x} \approx 1 - \frac{x}{2} - \frac{x^2}{4 \cdot 2!} - \frac{3x^3}{8 \cdot 3!}$$

$$\sqrt{\tfrac{1}{2}} = \sqrt{1-\tfrac{1}{2}} \approx 1 - \frac{(\frac{1}{2})}{2} - \frac{(\frac{1}{2})^2}{8} - \frac{(\frac{1}{2})^3}{16} = 0.7109$$

$$\sqrt{0.9} = \sqrt{1-.1} \approx 1 - \frac{.1}{2} - \frac{(.1)^2}{8} - \frac{(.1)^3}{16} = 0.9487$$

0.1 is closer to 0 than <u>0.5</u> is to 0. Therefore, the estimate for $\sqrt{0.9}$ will be more accurate.

<u>10.2</u>

$\boxed{1}$

$f(x) = \ln(x+2)$ $\qquad$ $f(2) = \ln 4$

$f'(x) = \dfrac{1}{x+2}$ $\qquad$ $f'(2) = \dfrac{1}{4}$

$f''(x) = \dfrac{-1}{(x+2)^2}$ $\qquad$ $f''(2) = \dfrac{-1}{16}$

$f'''(x) = \dfrac{2}{(x+2)^3}$ $\qquad$ $f'''(2) = \dfrac{1}{32}$

$$\ln(x+2) = \ln 4 + \frac{(x-2)}{4} - \frac{(x-2)^2}{16 \cdot 2!} + \frac{(x-2)^3}{32 \cdot 3!} + \cdots$$

$\boxed{2}$ $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$\boxed{3}$ $1+x+x^2+x^3+\cdots = \dfrac{1}{1-x}$

$$\frac{1}{1-x} = 7$$

$$\frac{1}{7} = 1-x$$

$$x = \frac{6}{7}$$

10.3

$\boxed{1}$ $\sin y = y - \dfrac{y^3}{3!} + \dfrac{y^5}{5!} - \dfrac{y^7}{7!} + \cdots$

Let $y = x^2$.

$$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots$$

$\boxed{2}$ $2x\cos(x^2) = \dfrac{d\left(\sin(x^2)\right)}{dx}$

$$2x\cos(x^2) = 2x - \frac{6x^5}{3!} + \frac{10x^9}{5!} - \frac{14x^{13}}{7!} + \cdots$$

$\boxed{3}$ $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$

$\dfrac{1}{(1-x)^2} = \dfrac{d}{dx}\left(\dfrac{1}{1-x}\right) = 1 + 2x + 3x^2 + 4x^3 + \cdots$

$\dfrac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \cdots$

<u>10.4</u>

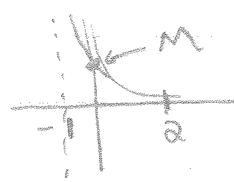
$\boxed{1}$ $f(x) = \sin x$

$f^{(4)}(x) \leq 1$ for $0 \leq x \leq 2$.

$|E_3| = |f(2) - P_3(2)| \leq \dfrac{|2|^4}{4!} = \dfrac{2}{3}$

$\boxed{2}$ $f(x) = (1+x)^{-1/2} = \dfrac{1}{\sqrt{1+x}}$, $f(2) = \dfrac{1}{\sqrt{3}}$

$f^{(4)}(x) = \dfrac{105}{16(1+x)^{9/2}}$



$M = \dfrac{105}{16}$

$|E_3| = |f(2) - P_3(2)| \leq \dfrac{105\,|2|^4}{16 \cdot 4!} = \dfrac{35}{8}$

$\boxed{3}$ By looking at a graph,

$P_2(\theta) = P_1(\theta) = \theta$ is an overestimate for $0 < \theta \le 1$.
$P_2(\theta) = P_1(\theta) = \theta$ is an underestimate for $-1 \le \theta < 0$.

$|f^{(3)}(\theta)| \le 1$ for $-1 \le \theta \le 1$

$$|E_2| \le \frac{1 \cdot |1|^3}{3!} = \frac{1}{6}$$

11.1

$\boxed{1}$ $y' = -9\sin(3t)$
$y'' = -27\cos(3t)$

$-27\cos(3t) + 9(3\cos(3t)) \stackrel{?}{=} 0$
$-27\cos(3t) + 27\cos(3t) \stackrel{?}{=} 0$
$0 = 0$ ✓

$\boxed{2}$ $y' = \omega e^{\omega t}$
$y'' = \omega^2 e^{\omega t}$

$\omega^2 e^{\omega t} - 16e^{\omega t} = 0$
$(\omega^2 - 16)\, e^{\omega t} = 0$
never 0

$\omega^2 - 16 = 0$
$\omega^2 = 16 \implies \omega = \pm 4$

$\boxed{3}$ $P''(t) = P'(t)(4 - P(t)) - P(t)P'(t)$

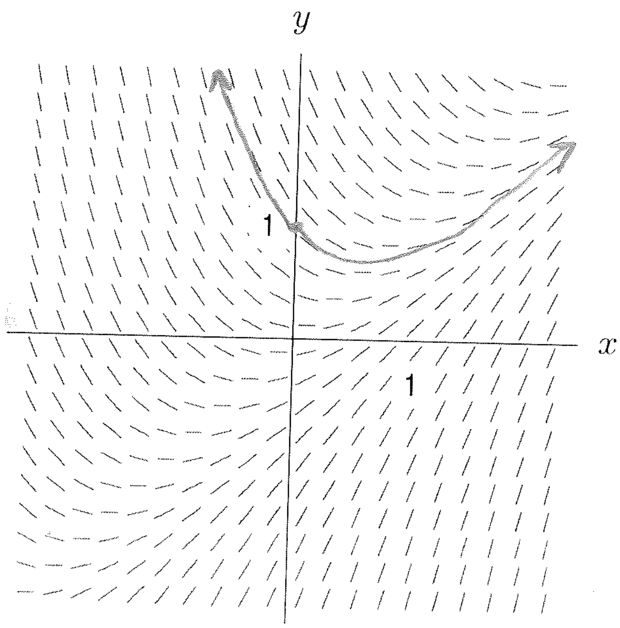
$P''(0) = P'(0)(4 - P(0)) - P(0)P'(0)$

$= 3(4 - 1) - 1(3)$

$= 6$

11.2

$\boxed{1}$



$y \approx 1.8$ when $x = 1$

$\boxed{2}$ $y = ax + b$

$y' = a$

$a \stackrel{?}{=} x - \frac{ax+b}{2}$

$a \stackrel{?}{=} x - \frac{a}{2}x - \frac{b}{2}$

$0x + a \stackrel{?}{=} (1 - \frac{a}{2})x - \frac{b}{2}$

$0 = 1 - \frac{a}{2}$ & $a = -\frac{b}{2}$

$\frac{a}{2} = 1$

$a = 2$ $\qquad 2 = -\frac{b}{2}$

$-4 = b$

$\Rightarrow y = 2a - 4$ is the unique solution which is a straight line.

3 This can be checked by a calculator or mathematica.

11.3

1

x	y	m	Δy
1	3	4	2
1.5	5	7.25	3.625
2	8.625	12.625	6.3125

$y \approx 8.625$

$$\frac{d^2y}{dx^2} = 2x + \frac{dy}{dx} = 2x + x^2 + y$$

$$\frac{d^2y}{dx^2} > 0 \quad @(1,3)$$

So the curve is concave up near (1,3). Therefore, the estimate will be an under-estimate.

2 $\frac{dy}{dx} = 0.5(2-y) = \frac{2-y}{2}$

x	y	m	Δy
0	1	$\frac{1}{2}$	$\frac{1}{6}$
$\frac{1}{3}$	$\frac{7}{6}$	$\frac{5}{12}$	$\frac{5}{36}$
$\frac{2}{3}$	$\frac{47}{36}$	$\frac{25}{72}$	$\frac{25}{216}$
1	$\frac{307}{216}$	$\frac{125}{432}$	$\frac{125}{1296}$

$y \approx \frac{307}{216}$

(9)

$\boxed{3}$ $\frac{dy}{dx} = x - y$

x	y	m	Δy
0	1	-1	$-\frac{1}{3}$
$\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{9}$
$\frac{2}{3}$	$\frac{5}{9}$	$\frac{1}{9}$	$\frac{1}{27}$
1	$\frac{16}{27}$	$\frac{11}{27}$	$\frac{11}{81}$

$y \approx \frac{16}{27} \approx .5925$

$1.8 > .5925$, this follows as an under-estimate since the curve is concave up.

<u>11.4</u>

$\boxed{1}$ $\frac{dQ}{dt} = 300 - 0.3Q$

$\frac{dQ}{300-0.3Q} = dt$

$\frac{-1}{.3}\int \frac{-.3\,dQ}{300-0.3Q} = \int dt$

$-\frac{1}{.3} \ln|300 - 0.3Q| = t + C$

$$\ln|300-.3Q| = -.3t + B$$

$$|300-.3Q| = e^{-.3t+B}$$

$$|300-.3Q| = e^{B}\cdot e^{-.3t}$$

$$300-.3Q = Ae^{-.3t}$$

$$.3Q = 300 - Ae^{-.3t}$$

$$Q = \frac{300-Ae^{-.3t}}{.3}$$

$$500 = \frac{300-A}{.3}$$

$$A = 150$$

$$Q(t) = \frac{300-150e^{-.3t}}{.3}$$

$\boxed{2}$

$$\frac{dy}{dx} = y^2+1$$

$$\int \frac{dy}{y^2+1} = \int dx$$

$$\arctan y = x + C$$

$$\arctan 1 = 0 + C$$

$$C = \frac{\pi}{4}$$

$$\arctan y = x + \frac{\pi}{4}$$

$$y = \tan\left(x+\frac{\pi}{4}\right)$$

$\boxed{3}$ $\frac{dy}{dx} = xy$

$\int \frac{dy}{y} = \int x\,dx$

$\ln|y| = \frac{x^2}{2} + C$

$|y| = e^{\frac{x^2}{2} + C}$

$|y| = e^{\frac{x^2}{2}} \cdot e^{C}$

$y = Be^{\frac{x^2}{2}}$

$5 = Be^{0}$

$5 = B$

$y = 5e^{x^2/2}$